CSE311 Microwave Engineering

LEC (07)
Transmission Lines_ Part III

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Chapter Contents

3.5 Voltage Standing Wave Ratio (VSWR)

$$V_s(z) = V_o^+[e^{-j\beta z} + \Gamma e^{j\beta z}]$$

- From Eqn. (3.45), the voltage on the line is the superposition of the incident wave $V_o^+ e^{-j\beta z}$ and the reflected wave $V_o^+ \Gamma e^{j\beta z}$.
- If the load is matched, $\Gamma = 0$, the magnitude of the voltage on the line is $|V(z)| = |V_0^+|$, which is constant. Such line is sometimes said to be "flat".
- When the load is mismatched, the presence of a reflected wave leads to a standing waves, where the magnitude of the voltage on the line is not constant. Thus from (3.45) we have:

3.5 Voltage Standing Wave Ratio (VSWR) (Continued)

$$V(z) = V_o^+[e^{-j\beta z} + \Gamma e^{j\beta z}] = V_o^+ e^{-j\beta z}[1 + \Gamma e^{j2\beta z}] \quad (3.49)$$

The magnitude of the voltage is given by:

$$|V(z)| = |V_o^+||1 + \Gamma e^{j2\beta z}| = |V_o^+||1 + |\Gamma| e^{j(\varphi + 2\beta z)}| \quad (3.50)$$

- Where φ is the phase of reflection coefficient, Γ, (3.44). At any distance (z = - ℓ) measured from the load (z=0), the voltage magnitude is given by

$$|V(-l)| = |V_o^+||1 + |\Gamma| e^{+j(\varphi - 2\beta l)}| \quad (3.51)$$

- This result (3.51) shows that the voltage magnitude oscillates with position (z = - ℓ) along the line as shown in Fig. 3.3.
- The maximum value occurs when the phase term $e^{j(\varphi - 2\beta l)} = 1$, and is given by:

$$V_{max} = |V_o^+||1 + |\Gamma|| \quad (3.52)$$

- The minimum value occurs when the phase term $e^{j(\theta - 2\beta l)} = -1$, and is given by:

$$V_{min} = |V_o^+||1 - |\Gamma|| \quad (3.53)$$

- From (3.50) and Fig. 3.3, it is seen that the distance between two successive voltage maxima (or minima) is:

$$2\beta l = 2\pi \quad or \quad l = \frac{\pi}{\beta} = \frac{\pi}{2\pi / \lambda} = \lambda / 2 \quad (3.54\ a)$$

3.5 Voltage Standing Wave Ratio (Continued)

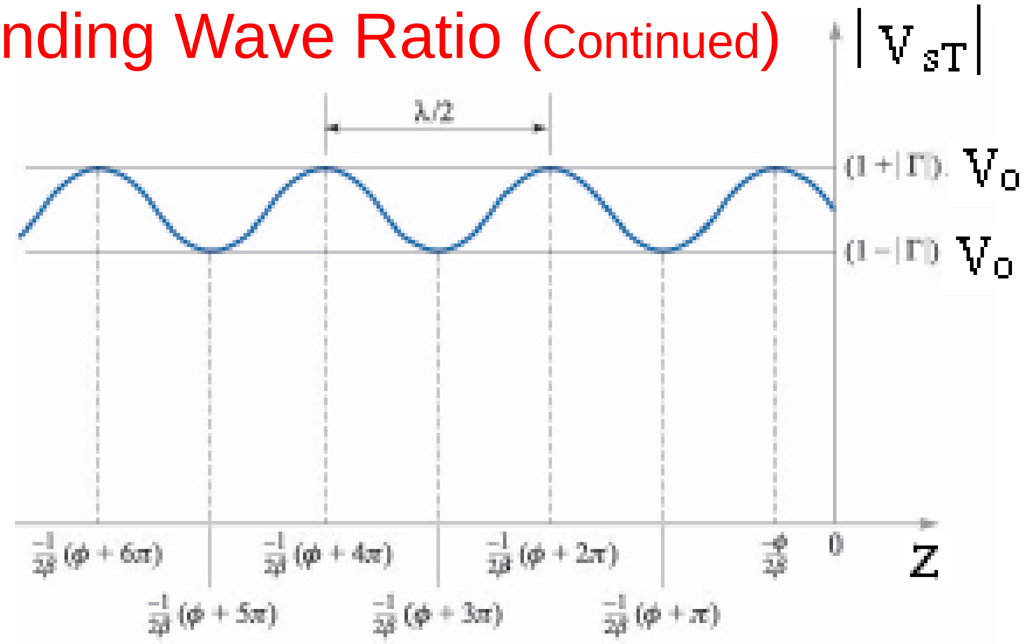


Fig.3.3 Voltage standing wave

- The distance between a maximum and a minimum is:

$$2\beta l = \pi \quad or \quad l = \lambda / 4 \qquad (3.54\ b)$$

- As $|\Gamma|$ increases, the ratio of V_{max} to V_{min} increases, so a measure of mismatch of a line called a standing Wave Ratio (SWR), can be defined as:

$$SWR = \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|} \qquad (3.55\ a)$$

- This quantity is also known as the voltage standing Wave Ratio (VSWR).
- SWR or VSWR is a real number such that $1 \leq SWR \leq \infty$, where SWR = 1 implies a matched load.

3.5 Voltage Standing Wave Ratio (VSWR) (Continued)

- Measuring the voltage standing Wave Ratio (VSWR) permits the immediate evaluation of $|\Gamma|$ from (3.55 a). Solving this equation for $|\Gamma|$, we get:

$$|\Gamma| = \frac{SWR - 1}{SWR + 1} \qquad (3.55\ b)$$

- Once Γ is known, the load impedance Z_L can be found in terms of Z_O by using (3.44).

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_o}{Z_L + Z_o} = |\Gamma| e^{j\varphi}$$

3.5 Voltage Standing Wave Ratio (VSWR) (Continued)

Example 3.6

A 50 Ω (Z_o = 50 Ω) lossless transmission line is terminated by a load impedance Z_L. If the a slotted line measurements yields: (VSWR = 5 and 15 cm spacing between two successive maxima). Calculate:

(a) The wavelength λ.
(b) The frequency f.
(c) The reflection coefficient Γ.
(d) The load impedance Z_L.

Solution:

(a) The spacing between two successive maxima is λ/2, then:

$\lambda/2 = 15$ cm   so, $\lambda = 30$ cm $= 0.3$ m

(b) The frequency, $f = c / \lambda = 3\text{x}10^8 / 0.3 = 1$ G Hz.

(c) From (3.55 b), we get:

$$|\Gamma| = \frac{SWR-1}{SWR+1} = \frac{5-1}{5+1} = \frac{2}{3}$$

(d) From (3.44), we get:

$$\Gamma = \frac{2}{3} = \frac{Z_L - Z_o}{Z_L + Z_o} \quad or \; Z_L = 5Z_o = 250\ \Omega$$

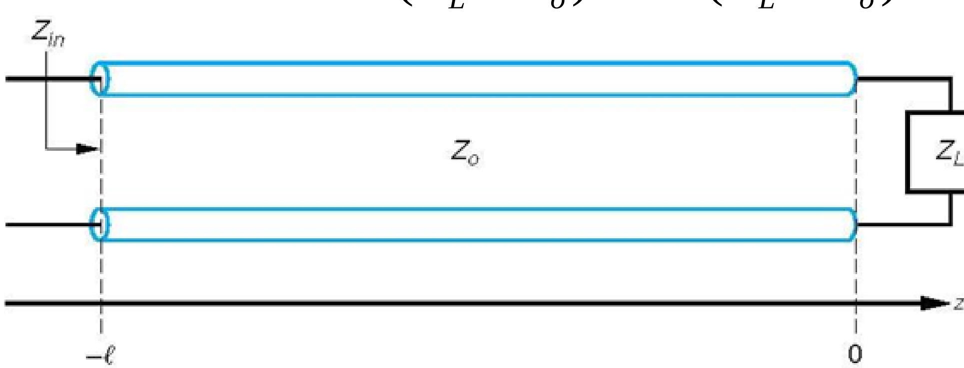
3.6 Transmission Line Input Impedance

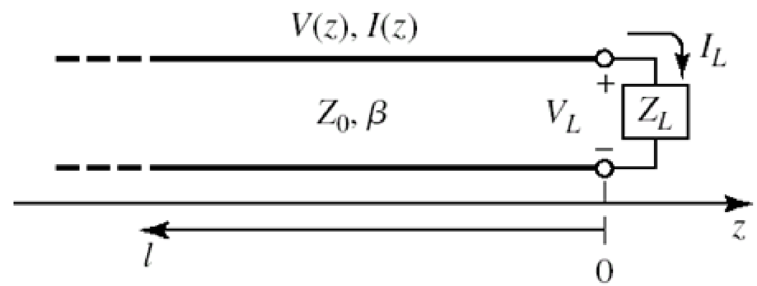
- The reflection coefficient (3.44) was defined as the ratio of the reflected to the incident voltage wave amplitudes at the load (z = 0).
- From (3.45) and (3.46), show that the voltage and current are oscillatory with position on the line.
- We may conclude that the impedance seen looking into the line must vary with position. $V_s(z) = V_o^+[e^{-j\beta z} + \Gamma e^{j\beta z}]$ $\quad I_s(z) = \frac{V_o^+}{Z_o}[e^{-j\beta z} - \Gamma e^{j\beta z}]$
- At a distance z = -*l* from the load, the input impedance seen looking toward the load is derived using (3.45) and (3.46) as:

$$Z_{in} = \frac{V(-l)}{I(-l)} = \frac{V_o^+[e^{j\beta l} + \Gamma e^{-j\beta l}]}{V_o^+[e^{j\beta l} - \Gamma e^{-j\beta l}]} Z_o = \frac{1 + \Gamma e^{-j2\beta l}}{1 - \Gamma e^{-j2\beta l}} Z_o \qquad (3.56)$$

Using (3.44) for Γ and substitute in (3.56), we get:

$$Z_{in} = Z_o \frac{(Z_L + Z_o)e^{j\beta l} + (Z_L - Z_o)e^{-j\beta l}}{(Z_L + Z_o)e^{j\beta l} - (Z_L - Z_o)e^{-j\beta l}}$$





3.6 Transmission Line Input Impedance

or $$Z_{in} = Z_o \frac{Z_L(e^{j\beta l} + e^{-j\beta l}) + Z_o(e^{j\beta l} - e^{-j\beta l})}{Z_o(e^{j\beta l} + e^{-j\beta l}) + Z_L(e^{j\beta l} - e^{-j\beta l})}$$

$$Z_{in} = Z_o \frac{Z_L \cos\beta l + jZ_o \sin\beta l}{Z_o \cos\beta l + jZ_L \sin\beta l} \qquad Z_{in} = Z_o \frac{Z_L + jZ_o \tan\beta l}{Z_o + jZ_L \tan\beta l} \qquad (3.57)$$

- This is an important result giving the input impedance of a length of transmission line with an arbitrary load impedance. (3.57) is referred to as the transmission line impedance equation.
- A number of special cases of lossless terminated transmission line will frequently appear in our work, so it is appropriate to consider the properties of some special cases as follows in the next section.

Example 3.7

The specifications per unit length of a certain transmission line operating at frequency, f = 200 MHz are: the series resistance, R = 20 mΩ/m, series inductance, L = 1 μH/m, shunt conductance, G = 8 μS/m and shunt capacitance, C = 0.4 nF/m. If the line is terminated by impedance, Z_L =100 + j75 Ω, as shown in Fig. 3.2, determine and calculate the following:

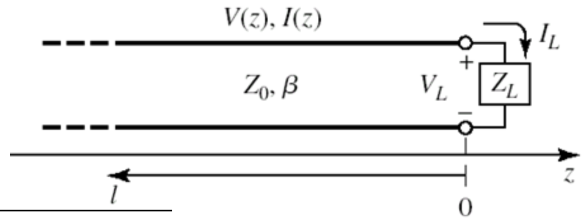
a) The wave parameters: λ, V_p and the characteristic impedance Z_o.

3.6 Transmission Line Input Impedance

Example 3.7

b) Calculate the reflection coefficient, Γ, the Voltage Standing Wave Ratio, VSWR and the input impedance (Z_{in}) if the length of the line is, *l* = 3 m. Knowing that is given by:

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan \beta l}{Z_o + jZ_L \tan \beta l}$$



Solution:

a)

$$\gamma = \sqrt{ZY} = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{([20x10^{-2} + j2\pi x200x10^{6} x1x10^{-6})x([8x10^{-6} + j2\pi x200x10^{6} x0.4x10^{-9})}$$

$$= \sqrt{(20x10^{-2} + j4\pi x x10^{2})x([8x10^{-6} + j16\pi x10^{-2})} = \sqrt{631.65\angle 179.99^o} = 25.13\angle 89.99^o$$

$$\gamma = \alpha + j\beta = 25.13\cos 89.88^o + j25.13\sin 89.88^o = 4.39 + j25.13 \quad \alpha = 4.39x10^{-3} \text{ NP/m} \quad \& \quad \beta = 25.13 \text{ rad/m}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{25.13} = 0.25 \ m \qquad V_p = \frac{\omega}{\beta} = \frac{2\pi x 20010^6}{25.13} = 0.5x10^8 \ m/\sec$$

$$Zo = \sqrt{\frac{R + j\omega L}{G + j\omega c}} = \sqrt{\frac{(20x10^{-2} + j4\pi x x10^{2})}{(8x10^{-6} + j16\pi x10^{-2})}} = 50 \ \Omega$$

b)

$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{100 + j75 - 50}{100 + j75 + 50} = 0.537\angle 29.74^o \quad SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.537}{1 - 0.537} = 3.32$$

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan \beta l}{Z_o + jZ_L \tan \beta l} = 50 \frac{100 + j75 + j50\tan(25.13x.3)}{50 + j(100 + j75)\tan(25.13x.3)l} = 50 \frac{100 + j75 + j50x3.07}{50 + j(100 + j75)x3.07l}$$

$$= 50 \frac{100 + j75 + j50x3.07}{50 + j(100 + j75)x3.07} = (35.65 - j19.19) \ \Omega$$

3.6 Transmission Line Input Impedance

Example 3.7 (Continued)

A source with source impedance, $Z_G = 50\ \Omega$ drives a 50 Ω transmission line that is 1/8 of wavelength long, terminated in a load $Z_L = 50 - j\,25\ \Omega$. Calculate:

a) The reflection coefficient, Γ_L.
b) Voltage Standing Wave Ratio, VSWR.
c) The input impedance, Z_{in} seen by the source.

Solution:

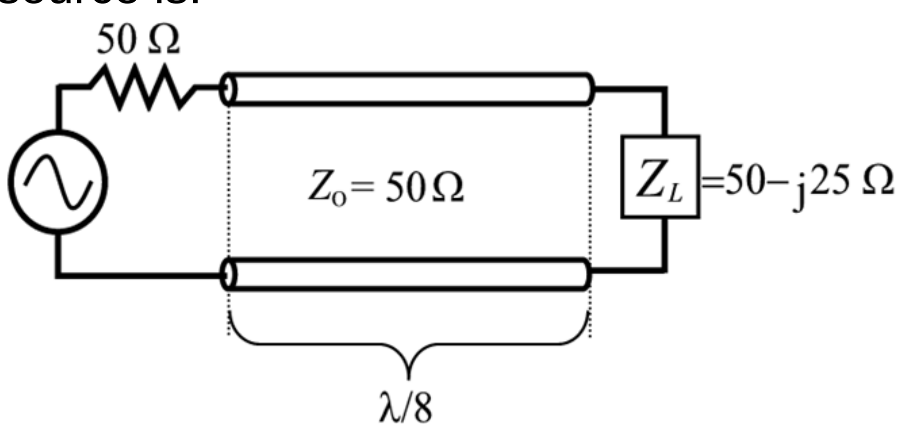
a) The reflection coefficient, Γ_L is:
$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(50 - j25) - 50}{(50 - j25) + 50} = 0.242e^{-j76^0}$$

b) Voltage Standing Wave Ratio, VSWR is:
$$VSWR = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = 1.64$$

c) The input impedance, Z_{in} seen by the source is:

since $\beta\ell = \frac{2\pi}{\lambda}\frac{\lambda}{8} = \frac{\pi}{4}$ $\quad \therefore \tan\frac{\pi}{4} = 1$

Therefore,
$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta\ell}{Z_0 + jZ_L \tan\beta\ell}$$
$$= 50\frac{50 - j25 + j50}{50 + j50 + 25}$$
$$= 30.8 - j3.8\Omega$$



3.6 Transmission Line Input Impedance (Continued)

3.6.1 Transmission Line Terminated in a Short Circuit

- Consider the transmission line circuit shown in Fig 3.5, where the line is terminated by a short circuit, $Z_L = 0$.
- From (3.44), the reflection coefficient is $\Gamma = -1$.
- From (3.55), the standing wave ratio is infinite (SWR = ∞).
- From (3.45) and (3.46), the voltage and current on the line are:

$$V(z) = V_o^+[e^{-j\beta z} - e^{j\beta z}] = -2jV_o^+ \sin \beta z \qquad (3.58)$$

$$I(z) = \frac{V_o^+}{Z_o} = [e^{-j\beta z} + e^{j\beta z}] = \frac{2V_o^+}{Z_o} \cos \beta z \qquad (3.59)$$

- At the load (Z = 0), (3.58) shows that V = 0 (as expected for a short circuit), while the current (3.59) is maximum (since cos $\beta Z = 1$).
- The input impedance $Z_{in} = V(-l)/ I(-l)$ or using (3.57) is given by:

$$Z_{in} = jZ_o \tan \beta l \qquad (3.60)$$

- Z_{in} is purely imaginary for any length, l. It takes on all values between $j\infty$ and $-j\infty$ as shown in Fig. 3.6. when $l = 0$ we have $Z_{in} = 0$, but for $l = \lambda/4$ we have $Z_{in} = \infty$.
- The impedance as indicated by (3.60) is periodic in l, repeating for multiples of $\lambda/2$.
- The voltage, current and impedance (reactance) variations along the short circuit transmission line are plotted in Fig.3.6.

3.6 Transmission Line Input Impedance (Continued)

3.6.2 Transmission Line Terminated in an Open Circuit

- Consider the transmission line circuit shown in Fig.3.7, where the line is terminated by an open circuit, Z = ∞.
- From (3.44), dividing the numerator and denominator by Z_L and allowing $Z_L \longrightarrow \infty$, the reflection coefficient is Γ = 1.
- From (3.55), the standing Wave Ratio is infinite (SWR = ∞)
- From (3.45) and (3.46), the voltage and current on the line are:

$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$SWR = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$V_s(z) = V_o^+[e^{-j\beta z} + \Gamma e^{j\beta z}] \quad \Longrightarrow \quad V(z) = V_o^+[e^{-j\beta z} + e^{j\beta z}] = 2V_o^+ \cos \beta z \qquad (3.61)$$

$$I_s(z) = \frac{V_o^+}{Z_o}[e^{-j\beta z} - \Gamma e^{j\beta z}] \Longrightarrow I(z) = \frac{V_o^+}{Z_o} = [e^{-j\beta z} - e^{j\beta z}] = \frac{-2jV_o^+}{Z_o} \sin \beta z \qquad (3.62)$$

- At the load (Z = ∞), (3.62) shows that the current, I = 0 (as expected for an open circuit), while the voltage (3.61) is maximum (since cos βZ = 1)
- The input impedance Z_{in} =V(-l)/ I(-l) or using (3.57) is given by:

$$Z_{in} = -jZ_o \cot \beta l \qquad (3.63)$$

- Z_{in} is purely imaginary for any length l. it takes on all values between j∞ and - j∞ as shown in Fig. 3.8. when l = 0 we have Z_{in} = -j∞, but for l = λ/4 we have Z_{in} = 0.
- The impedance as indicated by (3.63) is periodic in l, repeating for multiples of λ/2.
- The voltage, current and impedance (reactance) variations along the open circuit transmission line are plotted in Fig.3.8.